Développements limités usuels au voisinage de 0

(1) $\mathrm{e}^x = 1 + x + \dfrac{x^2}{2!} + \ldots + \dfrac{x^n}{n!} + x^n\varepsilon(x) = \displaystyle\sum_{k=0}^{n} \dfrac{x^k}{k!} + o(x^n)$

(2) $\cos x = 1 - \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + (-1)^n \dfrac{x^{2n}}{(2n)!} + o(x^{2n+1}) = \displaystyle\sum_{k=0}^{n} (-1)^k \dfrac{x^{2k}}{(2k)!} + o(x^{2n+1})$

(3) $\sin x = x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) = \displaystyle\sum_{k=0}^{n} (-1)^k \dfrac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$

(4) $\operatorname{ch} x = 1 + \dfrac{x^2}{2!} + \dfrac{x^4}{4!} + \cdots + \dfrac{x^{2n}}{(2n)!} + o(x^{2n+1}) = \displaystyle\sum_{k=0}^{n} \dfrac{x^{2k}}{(2k)!} + o(x^{2n+1})$

(5) $\operatorname{sh} x = x + \dfrac{x^3}{3!} + \dfrac{x^5}{5!} + \cdots + \dfrac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) = \displaystyle\sum_{k=0}^{n} \dfrac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$

(6) Pour α réel : $(1+x)^\alpha = 1 + \alpha x + \dfrac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \dfrac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + o(x^n)$

(7) $\sqrt{1+x} = 1 + \dfrac{1}{2}x - \dfrac{1}{2\times 4}x^2 + \dfrac{1\times 3}{2\times 4\times 6}x^3 + \cdots + (-1)^{n+1}\dfrac{1\times 3\times 5\times\cdots\times(2n-3)}{2\times 4\times\cdots\times 2n}x^n + o(x^n)$

(8) $\dfrac{1}{\sqrt{1+x}} = 1 - \dfrac{1}{2}x + \dfrac{1\times 3}{2\times 4}x^2 - \dfrac{1\times 3\times 5}{2\times 4\times 6}x^3 + \cdots + (-1)^n\dfrac{1\times 3\times 5\times\cdots\times(2n-1)}{2\times 4\times\cdots\times 2n}x^n + o(x^n)$

(9) $\dfrac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n)$

(10) $\dfrac{1}{1+x} = 1 - x + x^2 - \cdots + (-1)^n x^n + o(x^n)$

(11) $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} + \ldots + (-1)^{n-1}\dfrac{x^n}{n} + o(x^n)$

(12) $\tan x = x + \dfrac{x^3}{3} + \dfrac{2}{15}x^5 + x^6\varepsilon(x)$